

# MARS Technical Information

## Analysis of Multiphase Flow

### Governing Equations

The mass conservation equations for water (*w*) and NAPL (*o*) for incompressible liquids can be written as:

$$f \frac{\partial S_w}{\partial t} = - \frac{\partial q_{w_i}}{\partial x_i} \quad [1a]$$

$$f \frac{\partial S_o}{\partial t} = - \frac{\partial q_{o_i}}{\partial x_i} \quad [1b]$$

Darcy velocities in *p*-phase (*p* = *w*, *o* for water and oil phases) are defined by

$$q_{p_i} = -K_{p_{ij}} \left( \frac{\partial y_p}{\partial x_j} + \gamma_{rp} u_j \right) \quad [2]$$

where  $K_{p_{ij}}$  is the *p*-phase conductivity tensor,  $y_p = \frac{P_p}{g\gamma_w^*}$  is the water equivalent pressure head of phase

*p*,  $P_p$  is the *p*-phase pressure,  $g$  is gravitational acceleration, and  $\gamma_w^*$  is the density of pure water,  $\gamma_{rp} = \frac{\gamma_p}{\gamma_w^*}$

is the *p*-phase specific gravity,  $\gamma_p$  is the *p*-phase density, and  $u_j = \frac{\partial z}{\partial x_j}$  is a unit gravitational vector

measured positive upwards, with  $z$  as elevation,  $f$  is porosity and  $S_p$  is the *p*-phase saturation.

Integration of equation [2] in the  $z$  direction between the lower bound of the aquifer [ $z_l$ ] and upper bound equal to the top elevation of capillary fringe [ $z_u$ ] under the assumption of vertical equilibrium gives

horizontal, vertically integrated, fluxes  $\left[ \frac{L^2}{T} \right]$  as

$$Q_{wx_i} = -T_{wij} \frac{\int Z_{aw}}{\int x_j} \quad [3a]$$

$$Q_{ox_i} = -T_{oij} \frac{\int Z_{ao}}{\int x_j} \quad [3b]$$

where  $Q_{wx_i}$  and  $Q_{ox_i}$  are the horizontal vertically integrated fluxes  $[L^2T^{-1}]$  for water and oil, respectively in  $x_i (i=1,2)$  directions,  $T_{wij}$  is the water transmissivity tensor,  $T_{oij}$  is the oil transmissivity tensor (see Equation [6]).

The vertically integrated volume balance equation for water and oil phases can be expressed as

$$\frac{\int V_w}{\int t} = \frac{\int}{\int x_i} \left( T_{wij} \frac{\int Z_{aw}}{\int x_j} \right) + R_w \quad [4a]$$

$$\frac{\int V_o}{\int t} = \frac{\int}{\int x_i} \left( T_{oij} \frac{\int Z_{ao}}{\int x_j} \right) + R_o \quad [4b]$$

where  $V_w$  = specific volume [L] of water,  $V_o$  = specific volume [L] of NAPL expressed as

$$V_p = \int_{z_L}^{z_u} S_p dz \quad [5]$$

Phase transmissivities are expressed as

$$T_{p_{ij}} = \int_{z_L}^{z_u} k_{rp} K_{sp_{ij}} dz \quad [6]$$

where  $k_{rp}$  is the relative permeability of phase  $P$ ,  $K_{sw_{ij}}$  = saturated hydraulic conductivity tensor,

$K_{so_{ij}} = \frac{r_o K_{sw_{ij}}}{n_{ro}}$ ,  $z_u$  and  $z_L$  are the lower and upper limits of integration, and  $n_{ro}$  is the oil to water dynamic viscosities ratio.  $R_p$  is the source/sink rate  $\left(\frac{L}{T}\right)$  and  $P = w, o$  for water oil phases, respectively.

## **Residual NAPL Volume**

### Falling Air-Oil Table

When the air-oil,  $Z_{ao}$ , table falls, oil is trapped in the unsaturated zone. The maximum specific volume of oil trapped in the unsaturated zone is

$$V_{um} = S_{og} \phi \Delta Z_{ao} \quad [7]$$

where  $S_{og}$  is the maximum allowed unsaturated zone residual unsaturation and  $\Delta Z_{ao}$  is the change in the  $Z_{ao}$  elevation.  $S_{og}$  can be expressed as

$$S_{og} = F_g S_m (1-S_m) \quad [8]$$

where  $F_g$  may range from 0.2 to 0.5. Specific volume of trapped oil in the unsaturated zone  $V_u$  can be expressed as

$$V_u = \text{Min} (V_{um}, V_{ua}) \quad [9]$$

where  $V_{ua}$  is the actual specific volume of oil present in the unsaturated zone.

### Rising Water Table

When the oil-water table,  $Z_{ow}$ , rises, oil is trapped in the saturated zone. The maximum specific volume of oil trapped in the saturated zone can be expressed as

$$V_{sm} = S_{or} \phi \Delta Z_{ow} \quad [10]$$

where  $\Delta Z_{ow}$  is the change in the  $Z_{ow}$  elevation.  $S_{or}$  is the maximum allowed saturated zone residual saturation and can be expressed as

$$S_{or} = F_r (1-S_m) \quad [11]$$

where  $F_r$  ranges from 0.2 to 0.5. Specific volume of trapped oil in the saturated zone  $V_s$ , can be expressed as

$$V_s = \text{Min} (V_{sm}, V_{sa}) \quad [12]$$

where  $V_{sa}$  is the actual specific volume of oil present in the  $\Delta Z_{ow}$  section of unsaturated zone.

## **Calculation of Properties**

### Hook-Up Tables

Vertical distribution of water and oil saturations are computed and integrated for various values of product thickness using three phase pressure-saturation constitutive relations explained in the next section. Hook up tables are computed for fluid specific fluid (water and oil) volumes, fluid capacities, and transmissivities.

## Constitutive Relations

Relationships between phase saturations and pressures are described by a three-phase extension of the *van Genuchten*, 1980 model which takes into account effects of NAPL entrapment.

Prior to the occurrence of oil at a given location, the system is treated as a two-phase air-water system described by the *van Genuchten*, 1980 function

$$\bar{S}_w = \left[1 + (a y_{aw})^n\right]^{-m} \quad [13]$$

where  $\bar{S}_w = (S_w - S_m) / (1 - S_m)$  is the “effective” water saturation,  $S_m$  is the “irreducible” water saturation,  $a$  [ $L^{-1}$ ] and  $n$  [-] are porous medium parameters and  $m = 1 - \frac{1}{n}$ .

Following the occurrence of oil at a location, the system is described by the three-phase relations

$$\bar{\bar{S}}_w = \left[1 + (a b_{ow} y_{ow})^n\right]^{-m} \quad [14]$$

$$\bar{S}_t = \left[1 + (a b_{ao} y_{ao})^n\right]^{-m} \quad [15]$$

where  $\bar{\bar{S}}_w$  is the apparent water saturation,  $\bar{S}_t$  is the effective total liquid saturation,  $b_{ao}$  is a scaling coefficient which may be approximated by the ratio of water surface tension to oil surface tension and  $b_{ow}$  is a scaling factor approximated by the ratio of water surface tension to oil-water interfacial tension. The apparent water saturation is given by

$$\bar{\bar{S}}_w = \bar{S}_w + \bar{S}_{ot} \quad [16]$$

where  $\bar{S}_{ot}$  is the effective trapped oil saturation.

## Initial Conditions

$$Z_{aw}(x_i, 0) = Z_{aw}^o \quad [17a]$$

$$Z_{ao}(x_i, 0) = Z_{ao}^o \quad [17b]$$

Equations [17a] and [17b] describe the initial conditions for water and oil phases, respectively.

## Boundary Conditions

$$Z_{aw}(x_i, t) = Z_{aw1}(x_i) \quad t > 0 \quad \text{on } S_1 \quad [18a]$$

$$Z_{ao}(x_i, t) = Z_{ao1}(x_i) \quad t > 0 \quad \text{on } S_1 \quad [18b]$$

$$T_{w_{ij}} \frac{\nabla Z_{aw}}{\nabla x_i} = Q_{wf}(t) \quad t > 0 \quad \text{on } S_2 \quad [19a]$$

$$T_{o_{ij}} \frac{\nabla Z_{ao}}{\nabla x_i} = Q_{of}(t) \quad t > 0 \quad \text{on } S_2 \quad [19b]$$

Where equations [18a] and [18b] describe type-1 boundary conditions for water and oil phases, respectively, on section  $S_1$ , and equations [19a] and [19b] describe flux boundary condition for water and oil phases with vertically integrated normal fluxes  $\left[ \frac{L^2}{T} \right]$  for water and oil equal to  $Q_{wf}$  and  $Q_{of}$ , respectively, on region  $S_2$ .