

# MOVER Technical Information

## Analysis of Multiphase Flow

### Governing Equations

The mass conservation equations for water ( $w$ ) and NAPL ( $o$ ) for incompressible liquids and compressible gas can be written as:

$$f \frac{\partial S_w}{\partial t} = - \frac{\partial q_{w_i}}{\partial x_i} \quad [1a]$$

$$f \frac{\partial S_o}{\partial t} = - \frac{\partial q_{o_i}}{\partial x_i} \quad [1b]$$

$$f \frac{\partial r_a S_a}{\partial t} = - \frac{\partial r_a q_{a_i}}{\partial x_i} \quad [1c]$$

Darcy velocities in  $p$ -phase ( $p = w, o, a$  for water, oil, and gas phases) are defined by

$$q_{p_i} = -K_{p_{ij}} \left( \frac{\partial y_p}{\partial x_j} + r_{rp} u_j \right) \quad [2]$$

where  $K_{p_{ij}}$  is the  $p$ -phase conductivity tensor,  $y_p = \frac{P_p}{g r_w^*}$  is the water equivalent pressure head of phase  $p$ ,  $P_p$  is

the  $p$ -phase pressure,  $g$  is gravitational acceleration, and  $r_w^*$  is the density of pure water,  $r_{rp} = \frac{r_p}{r_w^*}$  is the  $p$ -

phase specific gravity,  $r_p$  is the  $p$ -phase density, and  $u_j = \frac{\partial z}{\partial x_j}$  is a unit gravitational vector measured positive

upwards and  $z$  is elevation,  $f$  is porosity and  $S_p$  is the  $p$ -phase saturation.

Integration of equation [2] in the  $z$  direction between the lower bound of the aquifer [ $z_l$ ] and upper bound equal to the top elevation of capillary fringe [ $z_u$ ] under the assumption of vertical equilibrium gives horizontal, vertically

integrated, fluxes  $\left[ \frac{L^2}{T} \right]$  as

$$Q_{wx_i} = -T_{w_{ij}} \frac{\partial Z_{aw}}{\partial x_j} \quad [3a]$$

$$Q_{ox_i} = -T_{oij} \frac{\nabla Z_{ao}}{\nabla x_j} \quad [3b]$$

$$Q_{ax_i} = -T_{a_{ij}} \frac{\nabla Z_{aa}}{\nabla x_j} \quad [3c]$$

where  $Q_{wx_i}$ ,  $Q_{ox_i}$ , and  $Q_{ax_i}$  are the horizontal vertically integrated fluxes [ $L^2 T^{-1}$ ] for water, oil, and gas, respectively in  $x_i$  ( $i = 1, 2$ ) directions,  $T_{wij}$  is the water transmissivity tensor,  $T_{oij}$  is the oil transmissivity tensor and  $T_{a_{ij}}$  is the gas transmissivity tensor (see Equation [6] later).

Vertically integrated volume balance equation for water, oil, and gas phases can be expressed as

$$\frac{\nabla V_w}{\nabla t} = \frac{\nabla}{\nabla x_i} \left( T_{wij} \frac{\nabla Z_{aw}}{\nabla x_j} \right) + R_w \quad [4a]$$

$$\frac{\nabla V_o}{\nabla t} = \frac{\nabla}{\nabla x_i} \left( T_{oij} \frac{\nabla Z_{ao}}{\nabla x_j} \right) + R_o \quad [4b]$$

$$\frac{\nabla V_a}{\nabla t} = \frac{\nabla}{\nabla x_i} \left( T_{a_{ij}} \frac{\nabla Z_{aa}}{\nabla x_j} \right) + R_a \quad [4c]$$

where  $V_w$  = specific volume [L] of water,  $V_o$  = specific volume [L] of NAPL and  $V_a$  = specific volume [L] of gas expressed as

$$V_w = \int_{Z_L}^{Z_u} S_w dz \quad V_o = \int_{Z_L}^{Z_u} S_o dz \quad V_a = \int_{P_{ao}}^{Z_s} S_a dz \quad [5]$$

Phase transmissivities for water and oil are expressed as

$$T_{p_{ij}} = \int_{Z_L}^{Z_u} k_{rp} K_{sp_{ij}} dz \quad [6a]$$

Phase transmissivity for gas is

$$T_{a_{ij}} = \int_{P_{ao}}^{Z_s} k_{ra} K_{so_{ij}} dz \quad [6b]$$

$k_{rp}$  is the relative permeability of phase  $P$ ,  $K_{swij}$  = saturated hydraulic conductivity tensor,  $K_{soij} = \frac{r_{ro} K_{swij}}{n_{ro}}$ ,

$K_{sa_{ij}} = \frac{r_{ra} K_{swij}}{n_{ra}}$ ,  $z_u$ , and  $z_l$  are the lower and upper limits of integration for water and oil phases,  $Z_s$  is ground surface elevation,  $n_{ro}$  is the oil to water dynamic viscosities ratio, and  $n_{ra}$  is the air to water dynamic viscosities ratio.  $R_p$  is the source/sink rate  $\left(\frac{L}{T}\right)$ ,  $P = w,o$  for water oil phases, and  $P_{ao}$  is the elevation of air-oil interface, respectively.

### **Residual NAPL Volume**

#### Falling Air-Oil Table

When the air-oil,  $P_{ao}$ , table falls, oil is trapped in the unsaturated zone. The maximum specific volume of oil trapped in the unsaturated zone is

$$V_{um} = S_{og} \phi \Delta P_{ao} \quad [7]$$

$$P_{ao} = Z_{ao} - h_a \quad [8]$$

where  $S_{og}$  is the maximum allowed unsaturated zone residual unsaturation and  $\Delta P_{ao}$  is the change in the  $P_{ao}$  elevation.  $S_{og}$  can be expressed as

$$S_{og} = F_g S_m (1-S_m) \quad [9]$$

where  $F_g$  may range from 0.2 to 0.5. Specific volume of trapped oil in the unsaturated zone  $V_u$  can be expressed as

$$V_u = \text{Min} (V_{um}, V_{ua}) \quad [10]$$

where  $V_{ua}$  is the actual specific volume of oil present in the unsaturated zone.

#### Rising Water Table

When the oil-water table,  $Z_{ow}$ , rises, oil is trapped in the saturated zone. The maximum specific volume of oil trapped in the saturated zone can be expressed as

$$V_{sm} = S_{or} \phi \Delta Z_{ow} \quad [11]$$

where  $\Delta Z_{ow}$  is the change in the  $Z_{ow}$  elevation.  $S_{or}$  is the maximum allowed saturated zone residual saturation and can be expressed as

$$S_{or} = F_r (1-S_m) \quad [12]$$

where  $F_r$  ranges from 0.2 to 0.5. Specific volume of trapped oil in the saturated zone  $V_s$ , can be expressed as

$$V_s = \text{Min} (V_{sm}, V_{sa}) \quad [13]$$

where  $V_{sa}$  is the actual specific volume of oil present in the  $\Delta Z_{ow}$  section of unsaturated zone.

## Calculation of Properties

### Hook-Up Tables

Vertical distributions of water and, oil and gas saturations are computed and integrated for various product thickness and gas vacuum values using three phase pressure-saturation constitutive relations explained in the next section. Hook up tables are computed for specific fluid (water, oil and gas) volumes, fluid capacities, and transmissivities.

### Constitutive Relations

Relationships between phase saturations and pressures are described by a three-phase extension of the *van Genuchten*, 1980 model which takes into account effects of NAPL entrapment.

Prior to the occurrence of oil at a given location, the system is treated as a two-phase air-water system described by the *van Genuchten*, 1980 function

$$\bar{S}_w = \left[ 1 + (a y_{aw})^n \right]^{-m} \quad [14]$$

where  $\bar{S}_w = (S_w - S_m) / (1 - S_m)$  is the “effective” water saturation,  $S_m$  is the “irreducible” water saturation,  $a [L^{-1}]$  and  $n [-]$  are porous medium parameters,  $m = 1 - \frac{1}{n}$ , and  $y_{aw} = h_a - y_w$  is the air-water capillary pressure.

Following the occurrence of oil at a location, the system is described by the three-phase relations

$$\bar{\bar{S}}_w = \left[ 1 + (a b_{ow} y_{ow})^n \right]^{-m} \quad [15]$$

$$\bar{S}_t = \left[ 1 + (a b_{ao} y_{ao})^n \right]^{-m} \quad [16]$$

where  $\bar{\bar{S}}_w$  is the apparent water saturation,  $\bar{S}_t$  is the effective total liquid saturation,  $y_{ow} = y_o - y_w$  is the oil-water capillary pressure,  $y_{ao} = h_a - y_o$  is air-oil capillary pressure,  $b_{ao}$  is a scaling coefficient which may be approximated by the ratio of water surface tension to oil surface tension and  $b_{ow}$  is a scaling factor approximated by the ratio of water surface tension to oil-water interfacial tension. The apparent water saturation is given by

$$\bar{\bar{S}}_w = \bar{S}_w + \bar{S}_{ot} \quad [17]$$

where  $\bar{S}_{ot}$  is the effective trapped oil saturation.

### Initial Conditions

$$Z_{aw}(x_i, 0) = Z_{aw}^o \quad [18a]$$

$$Z_{ao}(x_i, 0) = Z_{ao}^o \quad [18b]$$

$$Z_{aa}(x_i, 0) = Z_{aa}^* \quad [18c]$$

Equations [18a], [18b], and [18c] describe the initial conditions for water, oil, and gas phases, respectively.

#### Boundary Conditions

$$Z_{aw}(x_i, t) = Z_{aw1}(x_i) \quad t > 0 \text{ on } S_1 \quad [19a]$$

$$Z_{ao}(x_i, t) = Z_{ao1}(x_i) \quad t > 0 \text{ on } S_1 \quad [19b]$$

$$Z_{aa}(x_i, t) = Z_{aa1}(x_i) \quad t > 0 \text{ on } S_1 \quad [19c]$$

$$T_{w_{ij}} \frac{\partial Z_{aw}}{\partial x_i} = Q_{wf}(t) \quad t > 0 \text{ on } S_2 \quad [20a]$$

$$T_{o_{ij}} \frac{\partial Z_{ao}}{\partial x_i} = Q_{of}(t) \quad t > 0 \text{ on } S_2 \quad [20b]$$

$$T_{a_{ij}} \frac{\partial Z_{aa}}{\partial x_i} = Q_{af}(t) \quad t > 0 \text{ on } S_2 \quad [20c]$$

Where equations [19a], [19b], and [19c] describe type-1 boundary conditions for water, oil, and gas phases, respectively, on section  $S_1$ , and equations [20a], [20b], and [20c] describe flux boundary condition for water, oil, and gas phases with vertically integrated normal fluxes  $\left[ \frac{L^2}{T} \right]$  for water, oil, and gas equal to  $Q_{wf}$ ,  $Q_{of}$ , and  $Q_{af}$ , respectively, on region  $S_2$ .

#### Solution Techniques

Equations [4a] and [4b] for water and oil phases, respectively, are coupled and solved simultaneously using the Galerkin finite element method. The solution of gas phase equation [4c] is uncoupled with water and oil phases solution. MOVER uses 2-D rectangular or isoparametric elements to accurately model the physical boundaries and material boundaries.